一、 Determine the eigenvalues and eigenfunctions of the following differential equations (25%)

1. \( y'' + 2y' + \lambda y = 0 \), \( y(0) = 0, y(3) = 0 \)

2. \( x^2 y'' + xy' + \lambda y = 0 \), \( y(1) = 0, y(e^3) = 0 \)

二、 Consider the following heat-equation (25%)

\[
\frac{\partial u(x,t)}{\partial t} = 4 \frac{\partial^2 u(x,t)}{\partial x^2}
\]

with boundary conditions: \( u(0,t) = 0, u(4,t) = 0 \)

initial conditions: \( u(x,0) = \frac{3}{5} \sin 2\pi x + \frac{2}{5} \sin 4\pi x \)

三、 Consider the following Laplace equation (25%)

\[
\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0
\]

with boundary conditions: \( u(0,y) = 0, u(5,y) = g(y) \)

\( u(x,0) = 0, u(x,2) = f(x) \)

四、 Consider the following wave-equation (25%)

\[
\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial^2 u(x,t)}{\partial x^2}
\]

with boundary conditions: \( u(0,t) = 0, u(2,t) = 0 \)

initial conditions: \( u(x,0) = \frac{2}{5} \sin 2\pi x \)

\( u_t(x,0) = \frac{3}{2} \sin 6\pi x \)